QFT in curved spacetimes and analogue gravity
Lecture 3
Particle creation by black holes

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Contents

Historical orientation

- The frequency mixing mechanism introduced by Parker and its discovery of gravitational (cosmological) particle creation rapidly had a major impact on three main research groups:
  1. Moscow: Zeldovich's group

     1) Gravitational particle production in anisotropic cosmologies

     2) Superradiance in rotating black holes. Zeldovich and Starobinsky realized that a rotating object could amplify scalar waves. They interpreted this phenomenon as the classical analogue of a quantum process of stimulated emission

  2. Princeton: Wightman and Wheeler's groups

     1) Rindler space and the acceleration radiation (Fulling (1972), Unruh (1976))

     2) Extensive research on black holes and cosmology (Bekenstein, Hu, Unruh, Wald, Ford, ...)

  3. Cambridge

     Hawking's discovery of particle creation by black holes (1974)

Probability distribution of created particles by the expanding universe

- Let us go back to one of the most important results in Parker's analysis of (probability) distribution of created particles by the expansion of the universe (Lecture 2)
- The probability of creating one pair of particles, one with momentum $\vec{k}$ and the other with momentum $-\vec{k}$ $[(1(\vec{k}), 1(-\vec{k})]$ is

$$|_{out}\langle -\vec{k}, \vec{k}|0_{in}\rangle|^2 = \frac{|\beta_k|^2}{|\alpha_k|^2}|\langle 0_{out}|0_{in}\rangle|^2[\delta^3(\vec{0})]^2$$

- Vacuum persistence amplitude

$$\langle 0_{out}|0_{in}\rangle$$

- Another fundamental quantity is the relative probability of observing a pair of particles

$$\frac{|\beta_k|^2}{|\alpha_k|^2}$$

Thermal radiation

- Let us now focus on the relative probability $\frac{|\beta_k|^2}{|\alpha_k|^2}$. The typical large k behavior is

$$\frac{|\beta_k|^2}{|\alpha_k|^2} \sim_{k\to\infty} e^{-\mu k}$$

- Assume that this behavior happens for all k

$$\frac{|\beta_k|^2}{|\alpha_k|^2} = e^{-\mu\omega_k^{out}}$$

where μ is a constant parameter.

- Using $|\alpha_k|^2 - |\beta_k|^2 = 1$ one gets

$$n_k \equiv |\beta_k|^2 = \frac{\frac{|\beta_k|^2}{|\alpha_k|^2}}{1 - \frac{|\beta_k|^2}{|\alpha_k|^2}} = \frac{e^{-\mu\omega_k^{out}}}{1 - e^{-\mu\omega_k^{out}}} = \frac{1}{e^{\mu\omega_k^{out}} - 1}$$

Planck spectrum of (scalar) thermal radiation at the Temperature:

$$T = \mu^{-1}$$

- For fermions $|\alpha_k|^2 + |\beta_k|^2 = 1$. We then get for n_k a Planck distribution of thermal radiation obeying the Fermi-Dirac statistics $n_k \equiv |\beta_k|^2 = \frac{1}{e^{\mu\omega_k^{out}}+1}$.

From coherent superposition to incoherent mixture

- As shown in Lecture 2, the state $|0_{in}\rangle$ can be expressed as a coherent superposition of quantum states describing particle pairs at late times

$$
\begin{aligned}
|0_{in}\rangle &= \langle 0_{out}|0_{in}\rangle \prod_{\vec{k}} \exp\left(\frac{1}{2}\beta_k^* \alpha_k^{*-1} a^\dagger_{\vec{k}} a^\dagger_{-\vec{k}}\right)|0_{out}\rangle \\
&= \langle 0_{out}|0_{in}\rangle \prod_{\vec{k}} \exp\left(\frac{1}{2} e^{-\frac{\omega_k^{out}}{2T}} a^\dagger_{\vec{k}} a^\dagger_{-\vec{k}}\right)|0_{out}\rangle \\
&= \langle 0_{out}|0_{in}\rangle \prod_{[\vec{k}]} \sum_{n=0}^{\infty} e^{-\frac{n\omega_k^{out}}{2T}} |n(\vec{k})\rangle \otimes |n(-\vec{k})\rangle
\end{aligned}
$$

  We have absorbed the phases of $\beta_k^* \alpha_k^{*-1}$ in the creation operators.
- Note 1: It is an example of the so-called thermofield doble state. It arises naturally within this physical contex.
- Note 2: recall (in the discrete normalization)

$$
|n_j(\vec{k_j})\rangle = \frac{1}{\sqrt{n(\vec{k_j})!}} a^\dagger_{\vec{k_j}} |0_{out}\rangle \ , \quad |n_j(\vec{k_j}), n_j(-\vec{k}_j)\rangle = \frac{1}{n_j(\vec{k_j})!} a^\dagger_{\vec{k_j}} a^\dagger_{-\vec{k}_j} |0_{out}\rangle
$$

- But blackbody radiation is an incoherent mixture, described by a density matrix

$$\rho = Z^{-1} \sum_i e^{-E_i/T} |i\rangle\langle i| \ ,$$

where $|i\rangle$ are energy eigenstates.

- Taking the partial trace over particles with, for instance, $\vec{k}_z < 0$ (corresponding to modes with negative $\vec{k}_z$, which are assumed inaccesible to our detectors) we obtain an incoherent thermal mixture at temperature T
- The radiation is no longer described by a pure quantum state, but rather a thermal density matrix

$$|0_{in}\rangle \quad \to \quad \rho = |\langle 0_{out}|0_{in}\rangle|^2 \prod_{\vec{k}_z \geq 0} \sum_{n=0}^{\infty} e^{-\frac{n\omega_k^{out}}{T}} |n(\vec{k})\rangle\langle n(\vec{k})|$$

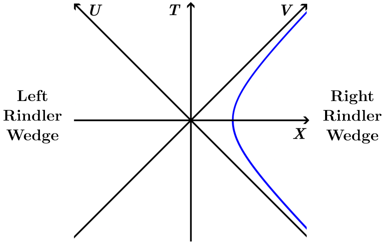
- Remark: this point is critical for understanding:
  1) Acceleration radiation (Fulling-Davis-Unruh effect)
  2) Thermal radiation from black holes (Hawking effect)

Fulling and the uniformly accelerated observer

- In his Ph.D. thesis, Fulling demonstrated that the superposition of creation and annihilation operators, similar to that introduced by Parker

$$a_{\vec{k}} = \alpha_k A_{\vec{k}} + \beta_k A^{\dagger}_{-\vec{k}} \tag{1}$$

also appears in the quantization of a field by uniformly accelerated observers



- To account for this situation, one needs to slightly generalize (1) to Bogoliubov transformations that involve a sum over different momenta, as in

$$a_{\vec{k}} = \sum_{\vec{k}'} \left[\alpha_{\vec{k},\vec{k}'} A_{\vec{k}'} + \beta_{\vec{k},\vec{k}'} A^{\dagger}_{-\vec{k}'} \right] .$$

Acceleration radiation

- Uniformly accelerated trajectory in Minkowski space

$$T = \frac{e^{a\xi}}{a} \sinh at \ , \ X = \frac{e^{a\xi}}{a} \cosh at$$

  The world line with $\xi = 0$ has constant acceleration a

- Metric:

$$ds^2 = -dT^2 + dX^2 + dY^2 + dZ^2 = -e^{2a\xi} dt^2 + d\xi^2 + dY^2 + dZ^2$$

- Consider a free scalar quantum field Φ

$$(\Box + m^2)\Phi = 0$$

Acceleration radiation

- Inertial (Minkowskian) Modes ($k_0 = \sqrt{m^2 + k_X^2 + \vec{k}_\perp^2}$)

$$f^M_{k_X, \vec{k}_\perp}(T, X, \vec{X}_\perp) = \frac{1}{\sqrt{2(2\pi)^3 k_0}} e^{-ik_0 T + i(k_X X + \vec{k}_\perp \vec{X}_\perp)} .$$

- Massless wave equation for the accelerated observer:

$$[(-\partial_t^2 + \partial_\xi^2) - e^{2a\xi}(m^2 + k_Y^2 + k_Z^2)]\Phi(t, \xi) e^{i\vec{k}_\perp \vec{X}_\perp} = 0$$

  like a scalar field in Minkowski space with the exponential potential $V(\xi) \propto e^{2a\xi}(m^2 + \vec{k}_\perp^2)$, where $\vec{k}_\perp^2 = k_Y^2 + k_Z^2$

- Accelerated Modes [$\kappa^2 \equiv m^2 + \vec{k}_\perp^2$]

$$g^R_{w, \vec{k}_\perp}(t, \xi, Y, Z) = \frac{e^{-iwt}}{2\pi^2 \sqrt{a}} \sinh^{\frac{1}{2}}\left(\frac{\pi w}{a}\right) K_{iw/a}\left(\frac{\kappa e^{a\xi}}{a}\right) e^{i\vec{k}_\perp \vec{X}_\perp}$$

See also: Crispino, Higuchi, Matsas, Rev. Mod. Phys. **80**, 787-838 (2008)

- Bogolubov coefficients [Fulling' 73, restricted to 1+1]

$$\beta_{w\vec{k}_\perp, k'_X \vec{k}'_\perp} = -\left[(e^{2\pi w/a} - 1)2\pi a k'_0\right]^{-1/2} \left(\frac{k'_0 + k'_X}{k'_0 - k'_X}\right)^{-iw/2a} \delta(\vec{k}_\perp - \vec{k}'_\perp) \ .$$

$$\alpha_{w\vec{k}_\perp, k'_X \vec{k}'_\perp} = \left[(1 - e^{-2\pi w/a})2\pi a k'_0\right]^{-1/2} \left(\frac{k'_0 + k'_X}{k'_0 - k'_X}\right)^{-iw/2a} \delta(\vec{k}_\perp - \vec{k}'_\perp) \ .$$

- Minkowski vacuum $|0_M\rangle \neq |0_R\rangle$ Accelerated (Rindler) vacuum
- This remarkable conclusion was first established in Fulling's doctoral thesis. However, the issue was somewhat confusing:

  This doesn't necessarily mean, however, that these eigenstates have anything to do with physical particles in the usual sense, things that trigger detectors and so on.
- Further clarification by P. Davies '75 (comparison with Hawking effect)

- Thermal result [Davies'75, in 1+1 dimensions]

$$\frac{|\beta_{w\vec{k}_\perp, k'_X \vec{k}'_\perp}|^2}{|\alpha_{w\vec{k}_\perp, k'_X \vec{k}'_\perp}|^2} = e^{-2\pi\omega/a}$$

- An uniformly accelerated observer perceives the Minkowski vacuum as a thermal bath of radiation at

$$T = \frac{\hbar a}{2\pi c k_B}$$

This is valid irrespective of the value of the mass m

- This result is crucially reinforced by Unruh's operationalism interpretation in terms of an accelerating particle detector

- Fulling '73- Davies '75- Unruh '76 Effect

Black hole radiation

Let us go back to 1973-74

- A central feature of Hawking's derivation was his treatment of black hole formation as a time-dependent process, an approach that closely paralleled Parker's analysis of a dynamically evolving universe.
- In the case of Schwarzschild black holes the analogue of the ratio in the expanding universe

$$\frac{|\beta_k|^2}{|\alpha_k|^2}$$

obtained in the far future, when the black hole has relaxed to a stationary configuration, and all transients effects have decayed, is (for simplicity, we omit the angular quantum numbers l, m)

$$\frac{|\beta_{\omega\omega'}|^2}{|\alpha_{\omega\omega'}|^2} = e^{-8\pi M\omega} \ .$$

- This yields a Planck distribution of thermal radiation at the (Hawking) temperature

$$T_H = \frac{\hbar c^3}{8\pi G M k_B} \ .$$

Hawking's famous discovery: historical curiosities

- This astonishing result was first submitted as a letter to Nature in January 1974.
- In February, Hawking presented his findings at the First Oxford Symposium on Quantum Gravity, held at Rutherford Laboratory.
- Around the same time, a Cambridge preprint containing a more detailed version of the calculations and results was circulated.
- The preprint was submitted to Communications in Mathematical Physics (CMP) in March, but the editors misplaced the manuscript, and Hawking had to resubmit it in April 1975.
- The paper was finally published in CMP in August 1975. The initial submission date was later corrected in a CMP erratum (1976).

Hawking's famous discovery: historical curiosities

- Hawking's CMP 1975 paper

Particle Creation by Black Holes

S. W. Hawking

Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge, England

Received April 12, 1975

- Erratum: CMP 1976

Erratum

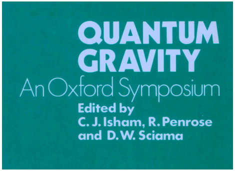
Hawking, S. W.: Particle Creation by Black Holes. Commun. math. Phys. **43**, 199—220 (1975)

Date of receipt: April 12, 1974

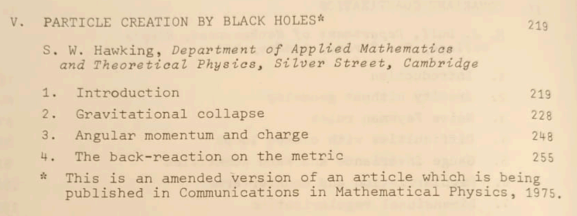
CMP.pdf

Hawking's famous discovery: historical curiosities

- Another interesting curiosity concerns the written version of Hawking's contribution at the Oxford Symposium, which appeared in the symposium proceedings also in August 1975.



- It was essentially an amended version of the article submitted to CMP.



V. PARTICLE CREATION BY BLACK HOLES* 219

S. W. Hawking, *Department of Applied Mathematics and Theoretical Physics, Silver Street, Cambridge*

* This is an amended version of an article which is being published in Communications in Mathematical Physics, 1975.

Hawking's famous discovery: historical curiosities

- The only difference was the inclusion of a citation to Parker's 1969 paper, added in the introduction at the end of the following paragraph:

  > *This means that the initial vacuum state $|0_1\rangle$, the state that satisfied $a_{1i}|0_1\rangle = 0$ for each initial annihilation operator a_{1i}, will not be the same as the final vacuum state $|0_3\rangle$ i.e., $a_{3i}|0_1\rangle \neq 0$. One can interpret this as implying that the time-dependent metric or gravitational field as caused the creation of a certain number of particles of the scalar field.*[(31)]

- Citation (31) corresponds to Parker's paper

Hawking's famous discovery: historical curiosities

- Citation (31) corresponds to Parker's paper

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(27) Teukolsky, S.A. Ap. J. 185 635-647 (1973).

(28) Unruh, W. Phys. Rev. Lett. 31 1265 (1973).

(29) Unruh, W. "Second Quantization in the Kerr Metric" Preprint 1974.

(30) Zeldovich, Ya. B. and Starobinsky, A.A. ZH.E.T.F. 61, 2161 (1971), JETP 34 1159 (1972).

(31) Parker, L. Phys. Rev. 183 1057 (1969).

- However, the omission of this citation in the final published version of Hawking's paper in CMP obscured the connection between Parker's earlier contributions [Lecture 2] and the approach Hawking employed
- A more comprehensive account is given in [A. Ferreiro, J. N-S and S. Pla, "The Birth of Gravitational Particle Creation ... ": [arXiv:2511.13518] To appear in The European Physical Journal H]

Derivation of black hole radiation

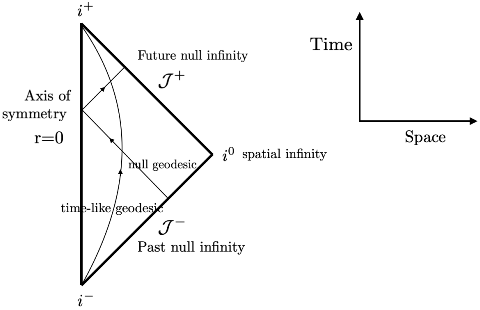
- We now derive black hole radiation using a set of simplified arguments
- These arguments are fully equivalent to the original method based on Parker's frequency-mixing mechanism

Background for deriving the Hawking effect

- As a preparation for explaining the derivation of the Hawking effect, we need to add additional background in two directions:
- 1) Geometrical aspects of the Schwarzschild black hole: Penrose diagrams, null geodesics, etc
- 2) Canonical quantization in spherical coordinates. Thermal field theory

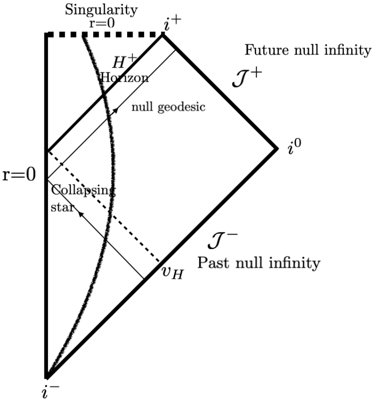
Penrose diagrams: Minkowski space

- A Penrose (conformal) diagram is a very convenient and useful representation of a spacetime, especially for those with spherical symmetry
- The Penrose diagram captures the causal structure of spacetime in a finite, planar diagram.It includes the infinities as boundaries of the diagram.
- Null geodesics are represented by lines at $\pm 45^{\circ}$
- Example 1: Penrose diagram of Minkowski space



Penrose diagrams: (spherical) Black hole formation

- Example 2: Penrose diagram describing the collapse of a star and black hole formation



Schwarzschild metric: radial null geodesics

- We need some extra background on the Schwarzschild metric and null geodesics
- Recall the Schwarzschild metric

$$ds^2 = \left(1 - \frac{2GM}{r}\right) dt^2 - \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 - r^2 d\Omega^2$$

where $d\Omega^2 \equiv d\theta^2 + \sin^2\theta \, d\varphi^2$ is the standard metric of the unit 2-sphere.

- We introduce the Regge-Wheeler or tortoise coordinate r^* by the equation

$$\frac{dr^*}{dr} = (1 - \frac{2GM}{r})^{-1} \, .$$

- The equations for the ingoing and outgoing radial null geodesics takes a simple form

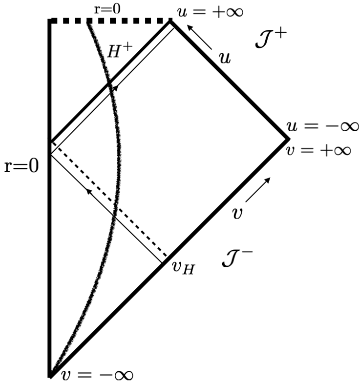
$$d(t \pm r^*) = 0$$

- Defining $v \equiv t + r^*$ an ingoing radial null geodesic is characterized by fixing the value of the coordinate $v = constant$

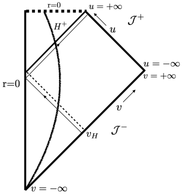
- One can also introduce the outgoing radial null coordinate $u \equiv t - r^*$.
- In the (u, v, θ, φ) coordinates the metric takes the form

$$ds^2 = (1 - \frac{2GM}{r})dudv - r^2 d\Omega^2$$

- Future null infinity $\mathcal{I}^+$ is characterized by $v = +\infty$, while past null infinity $\mathcal{I}^-$ is given by $u = -\infty$. The event horizon is located at $u = +\infty$.



- A gravitational collapse is a time-dependent process. To derive particle creation we have to analyze the time-evolution of the field modes in the background geometry.
- The process is tied to the propagation of modes near the horizon, which acts as a powerful microscope for a distant observer in the future. As a result, the mass of the quantized field becomes irrelevant to mode propagation when the modes are traced backward in time toward the horizon
- We want to find a steady particle production phenomena, not transient. Therefore, we have to especially analyze the outgoing field modes at late times $u \to +\infty$



- Hawking realized that the problem is then reduced to analyze the behavior of ingoing null geodesics that begin at $\mathcal{I}^-$, in the vicinity of $v \approx v_H$, with $v < v_H$. The mass of the field is irrelevant in this region.
- These geodesics will pass through the center of the collapsing star ($r = 0$), reaching $\mathcal{I}^+$ at very late times $u \to +\infty$.

Radial null geodesics: propagation from $\mathcal{I}^-$ to $\mathcal{I}^+$

- The geodesics with $v > v_H$ will enter the collapsing star, and after crossing the event horizon H^+will reach the singularity. $v = v_H$ is the last geodesic, it will form the event horizon at $u = +\infty$.

- The time-evolution of the field modes is reduced to follow the evolution of those null geodesics [in the vicinity of $v \approx v_H$, with $v < v_H$.]

- The geodesic evolution implies that the affine interval $(v_H - v) \approx 0$ is related to the affine interval $u \sim +\infty$ by the expression

$$u(v) = -4GM \ln \frac{|v_H - v|}{C}$$

  where C is a positive constant that depends on the details of the collapse.

- Remark: The extreme blueshift/redshift encoded in the expression above is consistent with the arguments given earlier: (i) the field's mass becomes irrelevant near the horizon and when traced back to $\mathcal{I}^-$; and (ii) the geometric-optics approximation is justified for the propagation of the field modes.

The Hawking effect

- The vacuum at past null infinity $\mathcal{I}^-$ is the standard vacuum in Minkowski spacetime $|0_{in}\rangle$. It is characterized by the positive-frequency modes

$$\sim \frac{1}{r} e^{-i\omega(t+r)} Y_{lm}(\theta, \phi)$$

- The coefficient of each partial wave behaves essentially as a field mode of a 1+1 dimensional QFT[We ignore here the effective potential $V_l(r)$]
- Consider for simplicity a massless fermionic field in 1+1 dimensions [By doing so, we neglect the contribution of the gray-body factors]
- The action for this $1+1$ fermionic theory is

$$S_{fermion} = \int d^2x [\psi^+ \partial_- \psi^+ + \psi^- \partial_+ \psi^-]$$

where $\psi^\pm$ are real Weyl fields obeying the field equations $\partial_- \psi^+ = 0$ and $\partial_+ \psi^- = 0$. Note: the theory is equivalent to the 2D Ising model at criticality.

- The vacuum at past null infinity $\mathcal{I}^-$ is characterized the two-point function

$$\langle 0_{in}|\psi^+(v_1)\psi^+(v_2)|0_{in}\rangle = -\frac{1}{4\pi} \frac{1}{v_1 - v_2}$$

- The big question is: How is the $|0_{in}\rangle$ vacuum perceived at future null infinity $\mathcal{I}^+$?

- The change of variables induced by geodesic propagation

$$v - v_H = Ce^{-u/4GM} \ ,$$

the reflection at $r = 0$ (outside the horizon), and the transformation law of the correlation functions lead to

$$\langle 0_{in}|\psi^-(u_1)\psi^-(u_2)|0_{in}\rangle = +\frac{1}{4\pi}\frac{1}{v(u_1) - v(u_2)}\sqrt{\frac{dv}{du}(u_1)}\sqrt{\frac{dv}{du}(u_2)}$$

- Remark: The appearance of the square root reflects the fact that the fermionic field carries scaling dimension $1/2$.This can be seen directly from the conformal invariance of action $S_{fermion}$

- Therefore, the vacuum $|0_{in}\rangle$ is perceived at late times at $\mathcal{I}^+$ through its correlation function

$$\langle 0_{in}|\psi^-(u_1)\psi^-(u_2)|0_{in}\rangle = \frac{1}{4\pi}\frac{1}{4GM}\frac{1}{e^{(u_1-u_2)/8\pi GM} - e^{-(u_1-u_2)/8\pi GM}}$$

- This correlation function is antiperiodic in imaginary time

$$u \to u + 8\pi GMi$$

- This is the hallmark of a thermal state with temperature

$$T_H = \frac{1}{8\pi GM}$$

- Remarks: this derivation is somewhat parallel to the one given in [Fredenhagen and Haag, CMP 127, 273, 1990; Agullo, Navarro-Salas, Olmo and Parker, PRD 76 (2007) 044018]

- An alternative and quick derivation follows from the transformation law for the energy-momentum tensor of the fermionic field

$$\langle 0_{in}| : T_{uu} : |0_{in}\rangle(u)|_{\mathcal{I}^+} = \left(\frac{dv}{du}\right)^2 \langle 0_{in}| : T_{vv} : |0_{in}\rangle(v)|_{\mathcal{I}^-} - \frac{1}{48\pi}\{v;u\} \ ,$$

where $\{v;u\}$ is the Schwarzian derivative, under the transformation

$$v - v_H = Ce^{-u/4GM}$$

- Assuming $\langle 0_{in}| : T_{vv} : |0_{in}\rangle(v)|_{\mathcal{I}^-} = 0$, we find

$$\langle 0_{in}| : T_{uu} : |0_{in}\rangle(u)|_{\mathcal{I}^+} = \frac{\pi}{24}\frac{1}{(8\pi GM)^2}$$

which corresponds a flux of (fermionic) thermal radiation at the temperature

$$T_H = \frac{1}{8\pi GM}$$

- For a comprehensive monograph see [Fabbri and Navarro-Salas, Modeling black hole evaporation, ICP-World Scientific, 2005]

The Hawking effect: general results

- Hawking radiation for bosonic/fermionic matter and for charged, rotating black holes

$$\frac{dN_{lmp}(w)}{dwdt} = \frac{1}{2\pi} \frac{\Gamma_{lmp}(w)}{e^{2\pi\kappa^{-1}(w - m\Omega_H - q\Phi_H)} \pm 1} ,$$

- Ω_H and Φ_H are the angular velocity and the electric potential of the black hole horizon. $\Gamma_{lmp}(w)$ are the gray-body factors
- The signs $\pm$ in the denominator account for the Bose or Fermi statistics.
- m, p and q are the corresponding axial angular momentum, helicity, and charge of the radiated particle.
- A rotating black hole preferentially emits particles with the same-sign angular momentum as the hole
- A charged black hole preferentially emits particles with the same-sign charge as the hole
- Hawking radiation tends to bring black holes to a Schwarzschild state

Final comments

- One of the major arguments supporting, and giving confidence, to the surprising phenomena of thermal black hole radiance is that it allows to recover a complete agreement between the physical behavior of black holes and the laws of thermodynamics.

- At the same time, this result opened a Pandora's box: the so-called information-loss problem

- If black holes radiate and eventually evaporate completely, an initially pure quantum state will end up as a mixed state, violating the basic principles of quantum mechanics.